

Consciousness as the Algebraic Quotient

Interface Boundary

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Abstract

We present a Kernel-Law framing of subjective experience in which consciousness is identified not with the full physical history of a system, but with the quotient structure induced by a localized observer algebra. In this view, neural, atomic, or field-level degrees of freedom reside primarily in the kernel of the observer map, while experienced states correspond to equivalence classes of histories that survive the induced quotient. This formalism provides structural explanations for the apparent ineffability of qualia, the privacy of experience, and the phenomenology of agency, without invoking non-physical substances or violations of underlying dynamics. Subjectivity emerges as a boundary condition imposed by finite, local access to an otherwise richer history space.

Keywords: consciousness; observer algebra; kernel quotient; qualia; subjectivity; free will

1 Setup

Let \mathcal{X} denote the full history space of a physical or computational substrate. An observer is characterized not by the substrate itself, but by an *observer algebra* \mathcal{O} , which determines what distinctions can be realized, measured, or reported.

Formally, the observer induces a map

$$\pi_{\mathcal{O}} : \mathcal{X} \rightarrow \mathcal{Y}_{\mathcal{O}},$$

where $\mathcal{Y}_{\mathcal{O}}$ is the space of accessible reports or observables.

This map induces an equivalence relation

$$x \sim_{\mathcal{O}} x' \quad \text{iff} \quad \pi_{\mathcal{O}}(x) = \pi_{\mathcal{O}}(x').$$

The *kernel* $\mathcal{K}_{\mathcal{O}}$ is the set of distinctions erased by this relation. The *quotient* $\mathcal{X}/\mathcal{K}_{\mathcal{O}}$ is the space of distinctions that survive access.

This quotient, rather than the full history, constitutes the experienced world.

2 Core Definition: “I Am the Algebra”

Standard physicalist approaches search for consciousness *inside* the history space \mathcal{X} : neurons, synapses, molecules, or fields. Kernel Law reframes the problem: the experienced world is not \mathcal{X} , but the quotient induced by the observer’s algebra.

Definition (Subject). A *subject* is an instanced observer algebra \mathcal{O}_{sub} together with its induced quotient $\mathcal{X}/\mathcal{K}_{\mathcal{O}_{sub}}$.

- **Unconscious (relative to \mathcal{O}):** the kernel $\mathcal{K}_{\mathcal{O}}$, containing all substrate activity that does not survive the observer map.
- **Conscious field:** the quotient $\mathcal{X}/\mathcal{K}_{\mathcal{O}}$, the space of resolved distinctions.
- **Experience:** the time-evolution of the quotient under coupled substrate and observer dynamics.

Interpretation. You are not the substrate “thing.” You are the cut—the algebra that separates signal from noise.

3 Qualia as Equivalence Classes

Why does “red” feel like something?

In Kernel Dynamics, “red” is not a wavelength label inside \mathcal{X} ; it is an element of the quotient $\mathcal{X}/\mathcal{K}_{\mathcal{O}}$.

Mechanism. The observer algebra \mathcal{O} maps a vast collection of distinct micro-histories

$$\{x, x', x'', \dots\} \subset \mathcal{X}$$

to a single quotient element $[x]_{\mathcal{O}}$.

Ineffability. The observer has no access to the internal members of $[x]_{\mathcal{O}}$. It has access only to the quotient label itself. Inspecting the internal structure would require an algebra expansion, replacing the original experience with a different one.

Conclusion. Qualia are compression artifacts of finite, localized algebras operating on richer histories.

4 Privacy of Experience

Why can one not feel another’s pain?

Not merely because bodies are separate, but because observer algebras differ:

$$\mathcal{O}_{me} \neq \mathcal{O}_{you}.$$

Proposition (Algebraic Privacy). Even with identical substrates, distinct causal positions and access constraints induce non-isomorphic observer algebras and therefore non-isomorphic quotients.

- **My pain:** a high-dimensional persistent object in $\mathcal{X}/\mathcal{K}_{me}$.
- **Your pain (to me):** a low-dimensional report in $\mathcal{X}/\mathcal{K}_{me}$.
- **Boundary:** merging algebras would eliminate the subject boundary, producing a different subject rather than shared experience.

Interpretation. Subjectivity is an algebraic boundary condition.

5 Agency as Kernel Uncertainty

The sensation of choice arises from finite access rather than indeterminism.

- The substrate history \mathcal{X} may evolve deterministically or unitarily.
- The observer algebra $\mathcal{O}(t)$ is finite and local.
- Future-determining distinctions therefore lie in $\mathcal{K}_{\mathcal{O}(t)}$.

Mechanism. A future choice outcome is a fact about \mathcal{X} but not yet a fact in the quotient. Deliberation is the computation that transfers distinctions from kernel to quotient.

Phenomenology. Relative to accessible variables, the future is open.

Slogan. Free will is the sensation of the kernel shrinking into the quotient.

6 The Subjective “Now”

- **Past:** stabilized structure in the quotient (memory).
- **Future:** unresolved kernel content.
- **Now:** the wavefront where kernel distinctions become experienced structure.

Informal consciousness equation.

$$\text{Consciousness} \sim \frac{d}{dt}(\mathcal{X}/\mathcal{K}_{\mathcal{O}(t)})$$

Final Line

We are not the record of the past, nor the full computation of the future. **We are the act of distinguishing.**