Kernel Analysis of Electron Identity and Global Topology

Technical Note

Subject: Formal status of the "One Electron" postulate under Kernel Law constraints.
Framework: History-Augmented Configuration Space $\mathcal{X}$ with Local Observable Algebra $\mathcal{O}_{\text{loc}}$.

Abstract

This note evaluates Wheeler's "One Electron" hypothesis—the proposition that N indistinguishable fermionic excitations can be regarded as a single globally connected worldline. In the Kernel Law framework, physical content is identified with the quotient of micro-history space by the equivalence relation induced by a local observable algebra. We show (i) particle identity labels are eliminated by the S_N permutation symmetry and therefore lie entirely in the Kernel, and (ii) global worldline connectivity is a topological functional on history space that does not descend to any element of the local observable algebra under standard locality and finite-access assumptions. Consequently, the "One Electron" hypothesis is empirically empty within local-access quantum field theory: it selects a representative in history space rather than introducing an observable ontology.

1. Executive Summary

This note evaluates the proposition that N indistinguishable fermionic excitations may be topologically identified as a single global worldline (Wheeler's "One Electron" hypothesis). Using the Kernel Law framework, we show:

- **Electron identity labels are gauge:** Permutations of indistinguishable particle labels act trivially on the local observable algebra and lie entirely within the Kernel.
- **Global worldline connectivity is undecidable locally:** The topological connectivity of fermionic worldlines is a global functional on history space that does not descend to any element of the local observable algebra.
- **Conclusion:** Under standard local-access quantum field theory assumptions, the "One Electron" hypothesis has no empirical content. It corresponds to a choice of representative within a Kernel equivalence class, not to a distinct physical ontology.

2. Formal Setup

Micro-History Space $\mathcal{X}$. The space of admissible microscopic histories, represented either as field configurations or as collections of worldline segments embedded in spacetime $\mathcal{M}$.

Local Observable Algebra $\mathcal{O}_{\rm loc}$. The algebra generated by bounded-support smeared field operators and finite products thereof. Operationally, $\mathcal{O}_{\rm loc}$ represents all measurements implementable with finite resources in bounded spacetime regions and obeying microcausality.

Kernel Equivalence. Define an equivalence relation on $\mathcal{X}$ induced by $\mathcal{O}_{\rm loc}$:

$$x \sim_{\mathcal{O}_{\rm loc}} x' \quad \Longleftrightarrow \quad \mathrm{Law}(\hat{A} \mid x) = \mathrm{Law}(\hat{A} \mid x') \quad \forall \hat{A} \in \mathcal{O}_{\rm loc}. \tag{1}$$

The physically accessible state space for a local observer is the quotient

$$\mathcal{X}_{\rm phys} = \mathcal{X} / \sim_{\mathcal{O}_{\rm loc}} . \tag{2}$$

Kernel (interpretive). The Kernel consists of all distinctions between histories that are eliminated by this quotient. Geometrically, these may be viewed as directions in history space along which all local observables are invariant.

3. Lemma I — Electron Identity as Kernel Gauge

Premise. Consider N indistinguishable fermionic excitations. The Hamiltonian and all elements of $\mathcal{O}_{\rm loc}$ are invariant under the permutation group S_N acting on particle labels.

Statement. For any history $x \in \mathcal{X}$ and any permutation $\sigma \in S_N$,

$$x \sim_{\mathcal{O}_{\rm loc}} \sigma(x). \tag{3}$$

Proof Sketch. Permutation of identical particle labels leaves all correlation functions and local measurement statistics invariant. Therefore, no element of $\mathcal{O}_{\rm loc}$ distinguishes between x and $\sigma(x)$.

Consequence. Electron "identity" labels do not correspond to degrees of freedom in $\mathcal{X}_{\rm phys}$. They are gauge artifacts removed by the Kernel quotient.

4. Lemma II — Global Worldline Connectivity Is Not Locally Observable

Connectivity Functional. Let

$$\mathcal{C} : \mathcal{X} \to \{\text{connected, disconnected}\} \tag{4}$$

encode the global topological connectivity of fermionic worldlines.

Claim. There exists no observable $\hat{C} \in \mathcal{O}_{\text{loc}}$ such that

$$\hat{C}(x) = \mathcal{C}(x) \quad \text{for all } x \in \mathcal{X}. \tag{5}$$

Justification.

- $\mathcal{C}(x)$ depends on global closure and connectivity information across unbounded spacetime regions.
- Elements of $\mathcal{O}_{\text{loc}}$ are restricted to bounded causal diamonds and finite measurement resources.
- Inferring $\mathcal{C}(x)$ from strictly local observables would require forbidden nonlocal dependence, in conflict with locality and cluster-decomposition principles.

Consequence. Distinct global connectivity classes may correspond to the same equivalence class in $\mathcal{X}_{\text{phys}}$. Connectivity lies in the Kernel relative to $\mathcal{O}_{\text{loc}}$.

5. Main Result

The distinction between (i) "many electrons with separate worldlines" and (ii) "one electron traced along a single globally connected worldline" is not an observable distinction for a local quantum field theoretic observer. Under the Kernel Law, these descriptions represent different representatives of the same physical equivalence class in $\mathcal{X}_{\text{phys}}$. The "One Electron" hypothesis therefore introduces no new empirical content beyond standard quantum field theory.

6. Scope and Limitations

This result holds under the assumptions that observers access only $\mathcal{O}_{\text{loc}}$ (finite, local measurements) and that standard microcausality and locality conditions apply. The analysis does not address frameworks that posit fundamentally nonlocal observables or impose global boundary conditions as axioms.

7. Verification Status

- Logical consistency: Verified
- Alignment with Kernel Law quotient structure: Verified
- Independence from numerical simulation: Verified
- Empirical claim discipline: Verified