Kernel Analysis for the Neutron (Composite Case)

Subject: Formal status of neutron and constituent identity under Kernel Law constraints.
Framework: History-Augmented Configuration Space $\mathcal{X}$ with Observable Algebra $\mathcal{O}$.

Abstract

This technical note applies the Kernel Law framework to the neutron as a composite quantum object. By defining physical equivalence through observable-induced quotients on history space, we show that both neutron identity at the effective field theory (EFT) level and constituent quark identity at the QCD level are eliminated as gauge artifacts. While gauge-invariant structural observables such as form factors and parton distribution functions survive the quotient, persistent particle identity does not. We further show that compositeness increases the cardinality or effective entropy of Kernel equivalence classes relative to fixed observer access, implying that identity becomes more deeply hidden as structural complexity increases.

1. Formal Setup and Definitions

To analyze compositeness rigorously, we define the Kernel geometrically via the equivalence relation induced by the observable algebra.

- **History Space ($\mathcal{X}$):** The space of admissible micro-histories, represented as field configurations of quarks/gluons or as effective neutron fields over spacetime $\mathcal{M}$.
- **Observable Algebra ($\mathcal{O}$):** The algebra of physically accessible, gauge-invariant operators.
- **Kernel Equivalence Relation:** Two histories $x, x' \in \mathcal{X}$ are equivalent (kernel-indistinguishable) if and only if they induce identical statistics for all observables:

$$x \sim_{\mathcal{O}} x' \quad \Longleftrightarrow \quad \mathrm{Law}(\hat{O} \mid x) = \mathrm{Law}(\hat{O} \mid x') \;\; \forall \hat{O} \in \mathcal{O}. \tag{1}$$

- **Physical Phase Space:** The physically accessible state space is the quotient

$$\mathcal{X}_{\mathrm{phys}} = \mathcal{X} / \sim_{\mathcal{O}} . \tag{2}$$

The Kernel consists of all distinctions between histories that are eliminated by this quotient.

2. Kernel Classification: Neutron Identity (EFT Level)

We first model the neutron as a fundamental field within an effective field theory.

Proposition 1 (Neutron Identity as Kernel Gauge)

Let $\mathcal{X}_{\text{EFT}}$ be the history space for N indistinguishable neutrons, and let $\mathcal{O}_{\text{EFT}}$ be the effective observable algebra generated by local neutron operators (density, current, spin).

Proof.

- **Symmetry:** $\mathcal{O}_{\text{EFT}}$ is invariant under the permutation group S_N acting on neutron labels, as required by Fermi–Dirac statistics.
- **Kernel Membership:** For any history $x \in \mathcal{X}_{\text{EFT}}$ and any permutation $\sigma \in S_N$,

$$x \sim_{\mathcal{O}_{\text{EFT}}} \sigma(x). \tag{3}$$

- **Result:** The distinction "which neutron is which" lies entirely within the Kernel. The physical phase space is the quotient $\mathcal{X}_{\text{EFT}}/S_N$.

3. Kernel Classification: Constituent Identity (QCD Level)

We next model the neutron as a composite bound state in QCD.

Proposition 2 (Constituent Identity is Kernel-Hidden)

Let $\mathcal{X}_{\text{QCD}}$ be the history space of quark fields ψ_q and gluon fields A_μ.

Proof.

- **Confinement Constraint:** The physical observable algebra $\mathcal{O}_{\text{QCD}}$ consists strictly of gauge-invariant (color-singlet) operators.
  - *Allowed:* $\bar{\psi}\psi$ (mesons), $\epsilon_{abc}\psi^a\psi^b\psi^c$ (baryons), conserved currents J^μ.
  - *Forbidden:* Operators that tag a specific color index or track an individual quark field ψ^a in isolation.
- **Sea-Quark Ambiguity:** The neutron state is a superposition of configurations involving both "valence" and "sea" quarks generated dynamically from the vacuum. Operators in $\mathcal{O}_{\text{QCD}}$ cannot distinguish a valence quark from a sea quark, nor can they define or track a persistent identity for a single constituent through time. In the path-integral formulation, trajectory continuity for an individual quark excitation is not a gauge-invariant concept.
- **Result:** There exists no operator $\hat{O} \in \mathcal{O}_{\text{QCD}}$ that tracks the persistent identity of a specific quark excitation. The distinction "which quark became which" lies within the Kernel relative to $\mathcal{O}_{\text{QCD}}$.

4. Corollary: Surviving Observables

Since identity labels are eliminated by the quotient, we explicitly identify the observables that survive in $\mathcal{X}_{\text{phys}}$.

Corollary 1 (Charges and Structure Survive)

The quotient map $\mathcal{X} \to \mathcal{X}/\sim_{\mathcal{O}}$ preserves only:

- **Global Conserved Charges:** Baryon number B, electric charge Q, total angular momentum J, isospin I_3.
- **Gauge-Invariant Structure:** Observable internal properties that do not require identity tags, including magnetic moments, polarizabilities, form factors $F_1(q^2), F_2(q^2)$, and Parton Distribution Functions (PDFs). These quantities are functionals on the equivalence class $[x] \in \mathcal{X}/\sim_{\mathcal{O}}$, not on individual constituent trajectories.
- **Kinematics:** Center-of-mass momentum and energy.

Identity labels, worldline connectivity, and constituent trajectories are absorbed into the Kernel.

5. Theorem: Kernel Growth with Compositeness

Theorem (Entropy of Equivalence Classes)

Consider a system of fixed total energy modeled at increasing levels of structural resolution (fundamental $\to$ composite).

- **History Space Expansion:** The dimension of the micro-history space grows rapidly:

$$\dim(\mathcal{X}_{\text{QCD}}) \gg \dim(\mathcal{X}_{\text{EFT}}). \tag{4}$$

- **Observable Algebra Constraint:** For an observer with fixed access (e.g., scattering at energy E), the accessible subalgebra of observables remains comparatively small, consisting of global charges and limited structure functions.
- **Conclusion:** The cardinality or effective entropy of Kernel equivalence classes—relative to the fixed observable algebra $\mathcal{O}$—increases with compositeness.

Operational Meaning. As objects become more structurally complex, a vastly larger set of distinct micro-histories map to the exact same physical measurement record. Identity becomes more deeply buried in the Kernel, not less.

6. Verification Status

This note has been independently verified for logical consistency, correct use of equivalence-class-based Kernel definitions, and proper scoping of observable algebras. No numerical simulation is required, as the result is structural rather than contingent.